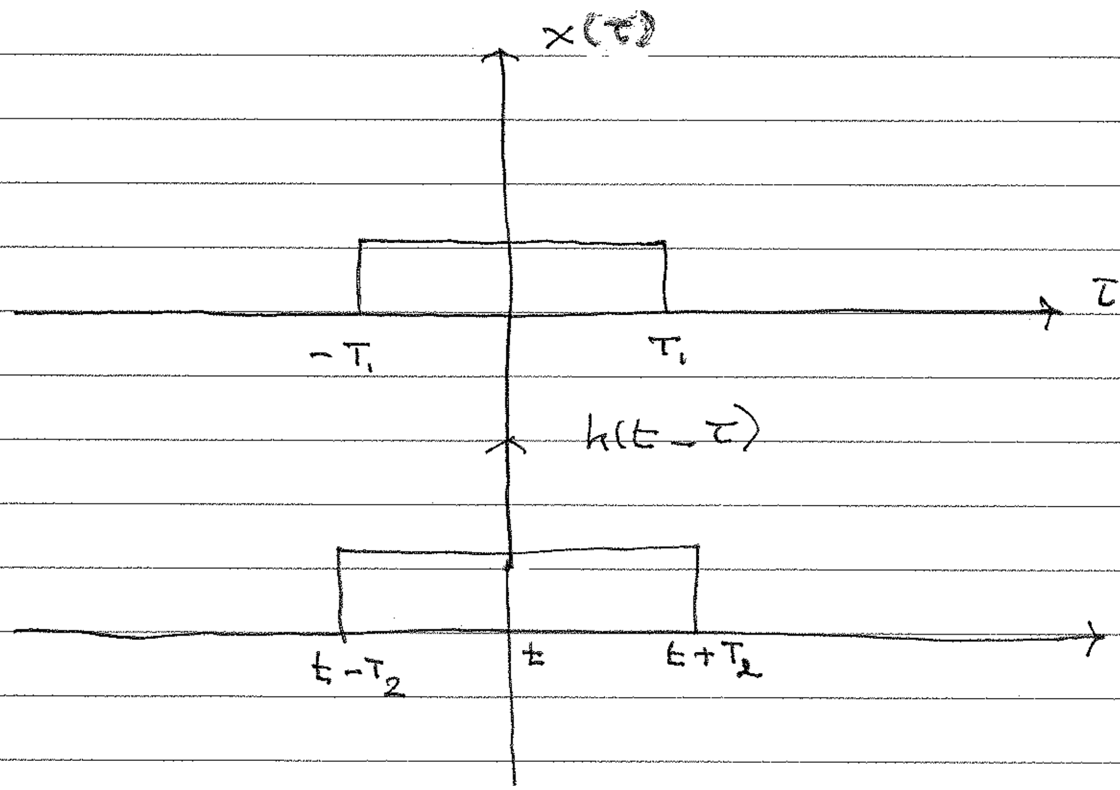
ECE-314, FALL 2018
SIGNALS & SYSTEMS

EXAMPLE : CONVOLUTION

(i) $$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & \text{otherwise} \end{cases}$$

$$h(t) = \begin{cases} 1, & |t| < T_2 \\ 0, & \text{otherwise} \end{cases}$$

Using $x(t)$ as the fixed signal and $h(t)$ as the sliding signal



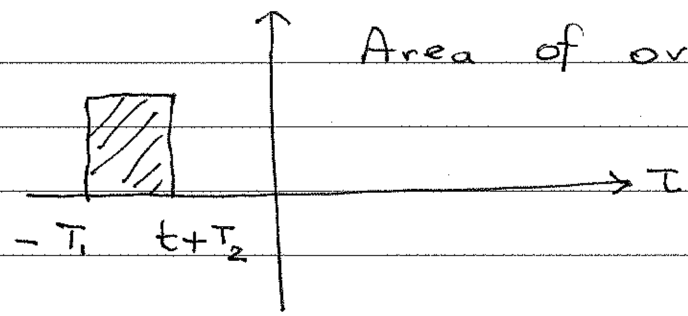
Assuming $T_1 > T_2$

For $t + T_2 < -T_1$ or $t < -T_1 - T_2$

(i) there is no overlap between the supports of $x(\tau)$, $h(t-\tau)$

$\Rightarrow \quad y(t) = 0, \quad t < -T_1 - T_2$

(ii) $t > -T_1 - T_2$ & $t + T_2 < 0$ or $t < -T_2$

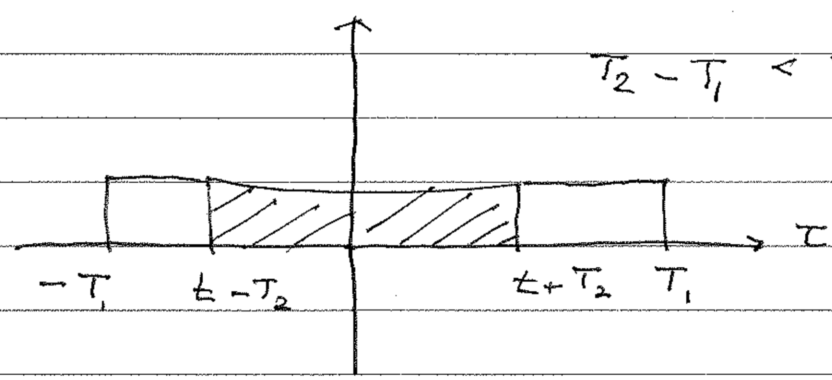


Area of overlap $=$ $(1)(t + T_2 + T_1)$

(iii) $t + T_2 > 0$

$\left.\begin{array}{l} t - T_2 > -T_1 \\ t + T_2 < T_1 \end{array}\right\}$

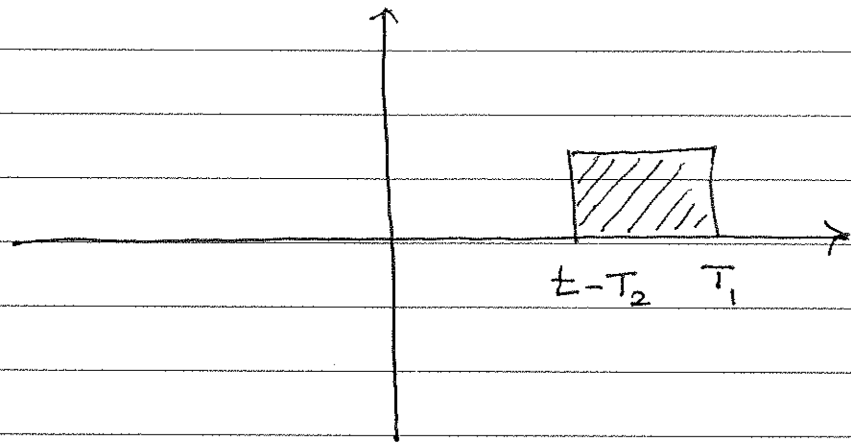
$T_2 - T_1 < t < T_1 - T_2$



Area of overlap : $2T_2$

$y(t) = 2T_2 \quad |t| < T_1 - T_2$

(iv) $t + T_2 > T_1$, $t - T_2 < T_1$ $\Big\}$ $t > T_1 - T_2$, $t < T_1 + T_2$



Area of overlap : $T_2 + T_1 - t$

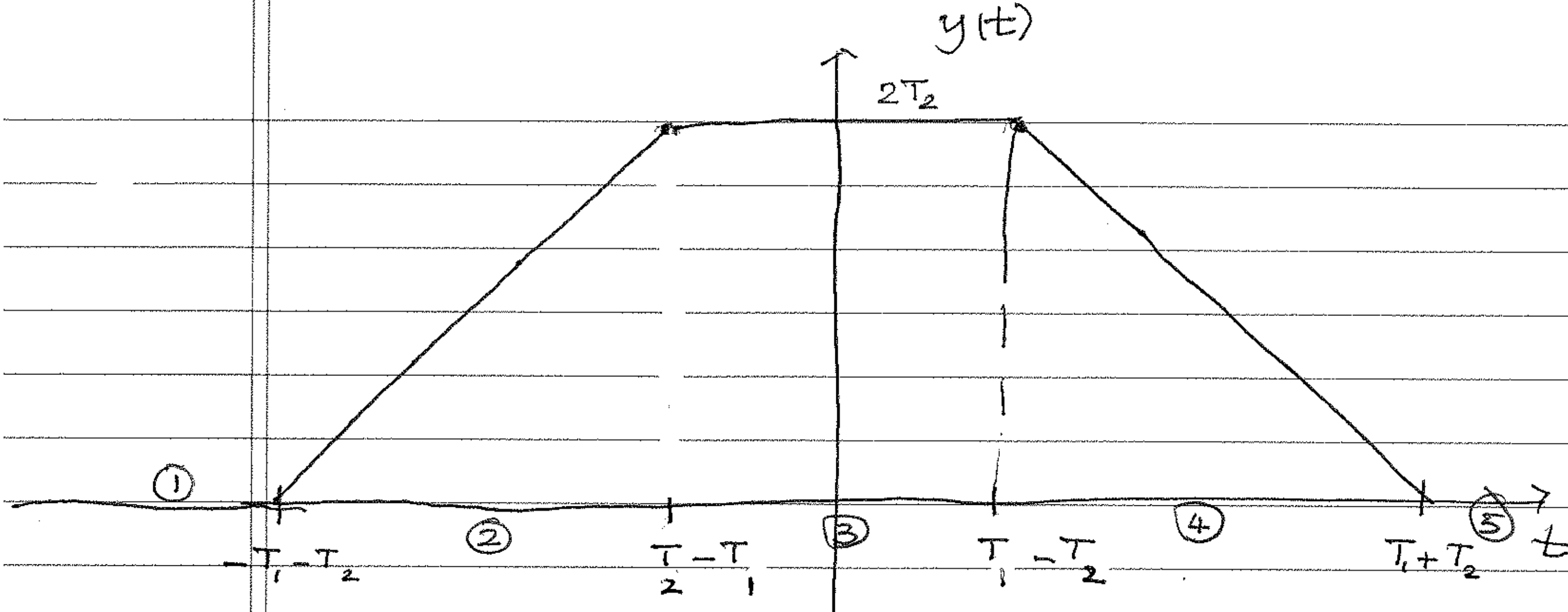
(v) $t > T_1 + T_2$

$x(\tau)h(t-\tau)$ do not overlap in support

$\Rightarrow \quad y(t) = 0, \quad t > T_1 + T_2$

Combining expressions :

$$y(t) = \begin{cases} 0, & |t| > T_1 + T_2 \\ T_1 + T_2 - |t|, & |t| > T_1 - T_2 \\ & |t| < T_1 + T_2 \\ 2T_2, & |t| < T_1 - T_2 \end{cases}$$



→ There are 5 different regions under consideration as depicted

→ In general difficult to keep track of regions analytically

→ Transform domain method solve problems in many cases

(ii) Discrete - time Convolution

$$x[n] = \begin{cases} 1, & n = 0 \\ 2, & n = 1 \\ 2, & n = 2 \\ 1, & n = 3 \\ 1, & n = 4 \\ 0, & \text{otherwise} \end{cases}$$

$$h[n] = \begin{cases} 3, & n = 0 \\ 2, & n = 1 \\ 1, & n = 2 \\ 0, & \text{otherwise} \end{cases}$$

Method 1

$$x[n] = \delta[n] + 2\delta[n-1] + 2\delta[n-2] + \delta[n-3] + \delta[n-4]$$

$$h[n] = 3\delta[n] + 2\delta[n-1] + \delta[n-2]$$

$$x[n] * h[n]$$

$$= 3\delta[n] * x[n] + 2\delta[n-1] * x[n] + \delta[n-2] * x[n]$$

$$
\begin{aligned}
y[n] = {} & 3\delta[n] + 6\delta[n-1] + 6\delta[n-2] \\
& + 3\delta[n-3] + 3\delta[n-4] \\
& + 2\delta[n-1] + 4\delta[n-2] + 4\delta[n-3] \\
& + 2\delta[n-4] + 2\delta[n-5] \\
& + \delta[n-2] + 2\delta[n-3] + 2\delta[n-4] \\
& + \delta[n-5] + \delta[n-6]
\end{aligned}
$$

Collecting identical terms:

$$
\begin{aligned}
y[n] = {} & 3\delta[n] + 8\delta[n-1] + 11\delta[n-2] \\
& + 9\delta[n-3] + 7\delta[n-4] \\
& + 3\delta[n-5] + \delta[n-6]
\end{aligned}
$$

This is the brute-force way of evaluating the convolution sum

Method 2:

$$
\underline{y}[n] \equiv \begin{pmatrix} y[0] \\ \vdots \\ y[7] \end{pmatrix} \triangleq \text{Output vector}
$$

$$
\underline{x}[n] \equiv \begin{pmatrix} x[0] \\ \vdots \\ x[7] \end{pmatrix} \triangleq \text{Input vector}
$$

INPUT SAMPLES $\longrightarrow$

$h[n-k]$	0	0	1	2	2	1	1	0	0
$n = 0$	$h[0]$	$h[1]$	$h[2]$	0	0	0	0	0	0
$n = 1$	0	$h[0]$	$h[1]$	$h[2]$	0	0	0	0	0
$n = 2$	0	0	$h[0]$	$h[1]$	$h[2]$	0	0	0	0
$n = 3$	0	0	0	$h[0]$	$h[1]$	$h[2]$	0	0	0
$n = 4$	0	0	0	0	$h[0]$	$h[1]$	$h[2]$	0	0
$n = 5$	0	0	0	0	0	$h[0]$	$h[1]$	$h[2]$	0
$n = 6$	0	0	0	0	0	0	$h[0]$	$h[1]$	$h[2]$

CONVOLUTION MATRIX $H_{7\times 8}$: (UPPER TRIANGULAR TOEPLITZ, CIRCULANT)

$\underline{y}_{7\times 1} = \mathbb{H}_{7\times 6}\, \underline{x}_{6\times 1}$ (CONVOLUTION $\equiv$ MATRIX MULTIPLICATION)